Coset realizations of (super)twistor spaces and structure of (super)twistor correspondence

A. T. Banin, N. G. Pletn'ev*

Institute of Mathematics, Novosibirsk,
Universitetsky prosp. 4, 630090, Russia

Abstract

New types "extended" (super)conformal algebras $G^{(\frac{n}{2})}$ are presented. (Super)twistor spaces T are subspaces in cosets $G^{(\frac{n}{2})}/H$. The (super)twistor correspondence has a cleary defined geometrical meaning.

*e-mail: pletnev@math.nsk.su

1 Introduction

Due to impressing applications of complex-analytic constructions to a number of physics problems, twistor theme makes an integral part of various disciplines. Famous papers [1] generated significant interest to this theme. Firstly, it was pointed out that equations of motion for massless fields on Minkowsky space M^4 with arbitrary spin could be rewritten as Cauchy – Riemann conditions for fields on twistor space T. In this way the problem of description of fields on M^4 comes to constructing of the space of complex structures on an oriented Riemann manifold and to the transformation of the original fields to the objects of complex algebraic geometry (fields on twistor space). It would like to apply twistor methods for the description of gravity as an essentially nonlinear theory. In the context of the twistor program the correspondence between conformal classes of auto-dual solutions of Einstein equations and deformations of functions in CP^3 was found in ref. [2]. There is analogous correspondence between the self-dual solutions of Yang-Mills (YM) equations and the two-dimensional fiber bundle over domains CP^3 [3], which leads to the classes of instanton and monopole solutions. This isomorphism is a mathematical design of the physics idea that the connection (as a dynamical field) shouldn't be defined on space-time points but on the ways (complex null-line), which are natural for the conformal invariant and holomorphic theories.

An additional interest in the selfduality connects with a proposition that all integrable systems can be gotten by the dimensional reduction from $4D$ self-dual conformal invariant theories [4]. The complexification of Minkowsky space is the common procedure for twistor methods. The inverse procedure — the realification of complex Minkowsky space is done by a sequential violation of $SL(4,C)$ symmetry. This procedure is connected with a choice of infinitely far light cone, a real structure and a scale.

Recently, it was arisen a tendency to use twistors for the solutions of basic problems of prepotential formulations of SUSY theories. An example, where SUSY and twistors are necessary for each other, is the conditions of integrability of wave equations on the light-like paths of superparticle interacting with gauge super YM fields and supergravity. These conditions define both the constrains and the equations of motion for SUSY formulations of such theories [5].

New "twistor-like" reformulations of the action principle for (super)particles and (super)strings [6, 7] turned out to be productive, because Cartan – Penrose conditions aren't postulated but arisen as the solutions of dynamical constrains and equations of motion. As this take place, Siegel k-symmetry [8] is a manifestation of local proper time SUSY, where the Grassmannian coordinate of "target" superspace and the momentum component of the twistor set up one supermuliplet. Number of the first-kind constrains in these models is sufficient to fix unphysical degrees of freedom. To get closed algebra "off–shell", a covariant form of the light-cone gauge is required. This gauge can be obtained through the use of some auxiliary variables which parameterize coset $SL\left(2,C\right)/H$, where H is some subgroup of $SL\left(2,C\right)$ [10].

It is well known, that the problems associated with the applying of the twistor approach to massive particles are arisen from the absence of conformal invariance of the action. Nevertheless, the using of holomorphic functions of several twistor variable allows us to find the twistor version of d'Alambert operator on twistor space. The simple calculation of the degrees of freedom shows that the twistor method has some excess parameters, which can be associated with some gauge symmetry. It is well known that gauge invariance

determines the main structure of any gauge theory. For one example, ever the theory of massless particle has $U(1)$ chiral symmetry that leaves kinematic twistors to be invariant.

According to Penrose, the natural symplectic structure of any complex algebraic manifold is based on a connection between the space-time description and the quantum-mechanical principle of superposition. This structure can be made consistent with the scheme of the canonical quantization.

In the given paper we present new types of (super)algebras which are related to the (super)twistor description. Usually, (super)twistors are considered as objects of a complex projective (super)space with a given action of a (super)conformal group. Nevertheless, basic twistor equation is invariant not only with respect to (super)conformal group but also with respect to so-named twistor shifts which affect the space-time coordinates. Extended with such shifts (super)conformal group will have the transitive action on spaces of flags.

We built straight "extensions" of the algebra of (super)conformal group through the addition of pairs of "twistor-like" generators. This approach is different from the tradition one, wherein the twistor manifolds are described by the cosets $SL\,(4,C)\,/P$, where P is some parabolic subgroup of $SL\,(4,C)$ (see for example ref.[9]). From our point of view the approach presented in this paper allows to find properties any twistor space more easily than the other known methods.

2 Constructions of the "extended" (super)algebras

2.1 Background and short description

Firstly, we present a number of common knowledges related to homogeneous spaces and to nonlinear realizations of groups.

Let $\mathcal{M}$ be a smooth manifold and G is some Lie group having differentiable action on $\mathcal{M}$, $G : \mathcal{M} \to \mathcal{M}$. Subgroup $H_p \subset G$ is called a stationary subgroup of a point $p \in \mathcal{M}$ iff $H_p : p = p$. If group G acts transitively on $\mathcal{M}$, i.e. $\mathcal{M}$ is a homogeneous manifold, then there is a projection $\rho(G/H) \to \mathcal{M}$, where $f \ni G/H$, $h \ni H$ and $f \sim fh$. When the structure of a smooth manifold can be introduced in G/H and ρ becomes a diffeomorphism. One say that manifold $\mathcal{M}$ is written in Klein form $\mathcal{M} = G/H$.

Let us consider the homogeneous space (coset) $F = G/H$. The group G is the full isometry group of F, and H is isotropy subgroup leaving the origin invariant. The coordinates in F are parameterized by group element $G \ni g = g(\phi_1 K_1, \phi_2 K_2, \ldots)$, where K_i denote the generators that are not in H. The manifolds G/H can also be thought of as sections of fiber bundles $\mathcal{F}$, with total space G and fiber H. These sections are parameterized by the group elements $f \in F$.

The main idea of nonlinear realizations stems from the fact that the isometries in G that are not in H are realized nonlinear on the fields ϕ_i in contrast the isometries in $H \subset G$. In this case one say that the symmetry is broken from G to H. It should be noted that the action of any subgroup $S \subset G$ on F will not be transitive. In general case we will have

$$F = \bigcup_k \mathcal{O}_k, \tag{1}$$

where $\mathcal{O}_k$ are orbits of the subgroup S. That also takes place when $S = H$.

It turns out that described above construction of the homogeneous spaces can be directly applied to the (super)twistors with some additional proposes. We will consider some

minimal extensions of the (super)conformal group which will be denoted by $G^{(\frac{n}{2})}$, where n will mark quantity of additional pairs of so-named "twistor-like" generators $(q_\alpha, s_{\dot\alpha})$. From one hand, any function on coset $G^{(\frac{n}{2})}/H$, where H is (super)conformal group, provides the representation[1] of H. But these functions will not be functions on the (super)twistor space in the ordinary sense because there is not the (super)twistor correspondence. From the other hand, we can get it through the consideration both of the coset $G^{(\frac{n}{2})}/\tilde H$, where $\tilde H \subset H$, and some invariant hypersurfaces $\tilde T \subset G^{(\frac{n}{2})}/\tilde H$ in this coset. The coset $G^{(\frac{n}{2})}/\tilde H$ will play the role so-called the correspondence space. We try to illustrate this thought some concrete examples below.

2.2 "Extended" algebra for the twistors

Let us present some examples. Usual conformal algebra $c(1,3) \sim su(2,2)$ with the commutation relation

$$\begin{aligned}
&[P_{\alpha\dot\alpha},\ K_{\beta\dot\beta}] = 2\varepsilon_{\alpha\beta}L_{\dot\alpha\dot\beta} + 2\varepsilon_{\dot\alpha\dot\beta}L_{\alpha\beta} + 4i\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}D,\\
&[L_{\alpha\beta},\ L_{\gamma\delta}] = \varepsilon_{\gamma(\beta}L_{\alpha)\delta} + \varepsilon_{\delta(\beta}L_{\gamma)\alpha},\\
&[L_{\dot\alpha\dot\beta},\ L_{\dot\gamma\dot\delta}] = \varepsilon_{\dot\gamma(\dot\beta}L_{\dot\alpha)\dot\delta} + \varepsilon_{\dot\delta(\dot\beta}L_{\dot\gamma)\dot\alpha},\\
&[D,\ P_{\alpha\dot\alpha}] = -iP_{\alpha\dot\alpha}, \qquad [D,\ K_{\alpha\dot\alpha}] = iK_{\alpha\dot\alpha},\\
&[L_{\alpha\beta},\ P_{\gamma\dot\gamma}] = \varepsilon_{\gamma(\beta}P_{\alpha)\dot\gamma}, \quad \mathit{and}\ \ (\alpha\beta \to \dot\alpha\dot\beta),\\
&[L_{\alpha\beta},\ K_{\gamma\dot\gamma}] = \varepsilon_{\gamma(\beta}K_{\alpha)\dot\gamma}, \quad \mathit{and}\ \ (\alpha\beta \to \dot\alpha\dot\beta).
\end{aligned} \tag{2}$$

where $L,\ P,\ D,\ K$ are Lorentz, momentum, dilation and special conformal generators respectively, we extend by one pair of generators $(q_\alpha, s_{\dot\alpha})$ with following nontrivial commutators

$$\begin{aligned}
&[P_{\alpha\dot\alpha},\ s_{\dot\beta}] = 2\varepsilon_{\dot\alpha\dot\beta}q_\alpha, \quad [K_{\alpha\dot\alpha},\ q_\beta] = -2\varepsilon_{\alpha\beta}s_{\dot\alpha},\\
&[L_{\dot\alpha\dot\beta},\ s_{\dot\gamma}] = \varepsilon_{\dot\gamma(\dot\beta}s_{\dot\alpha)}, \quad [L_{\alpha\beta},\ q_\gamma] = \varepsilon_{\gamma(\beta}q_{\alpha)},\\
&[D,\ q_\alpha] = -\tfrac{i}{2}q_\alpha, \qquad [D,\ s_{\dot\alpha}] = \tfrac{i}{2}s_{\dot\alpha}.
\end{aligned} \tag{3}$$

As a result we will have $G^{(\frac{1}{2})}$ "extended" algebra (which allows us to describe the twistor Z, but not $\bar Z$). This is a minimal extension of the conformal group. So, the conformal group action on the coset $G^{(\frac{1}{2})}/C(1,3)$ will be irreducible.

Now, we will illustrate how the equation, defining α – plane, can be produced from the outlined algebra. To get the twistor correspondence, let us define another coset F by the following choice[2]

$$F \ni f = \exp(\frac{i}{2}P_{\alpha\dot\alpha}x^{\dot\alpha\alpha})\exp(iq_\alpha\omega^\alpha + is_{\dot\alpha}\pi^{\dot\alpha}). \tag{4}$$

Here, we additionally introduced momentum generator in order to associate parameters $(\omega,\ \pi)$ with Minkowsky space coordinates $x \in M^4$ which can be considered as complex-valued. Particularly, the conformal group $C(1,3)$ will act nonlinearly on the parameters $x^{\dot\alpha\alpha}$. The other parameters $(\omega,\ \pi)$ will transform linearly under the conformal group action. From the latest follows that the correspondence will depend on the choice of the initial point in M^4. To extract in F subspaces that invariant under shifts of the twistor

[1] totally reducible in general case

[2] any other choice is also available, that will be represent redefinition of the group coordinates

coordinates $(\omega,\ \pi)$ we consider left-invariant Cartan's form on F. One, being restricted on F, is

$$f^{-1}df|_F = iq_\alpha(d\omega^\alpha + idx^{\dot\alpha\alpha}\pi_{\dot\alpha}) + is_{\dot\alpha}d\pi^{\dot\alpha} + iP_a dx^a. \tag{5}$$

It is easy to see that the conditions, extracting the subspaces,

$$d\omega^\alpha + idx^{\dot\alpha\alpha}\pi_{\dot\alpha} = 0, \quad d\pi_{\dot\alpha} = 0 \tag{6}$$

completely determine α – plane in proposition that $\omega = \omega(x),\ \pi = \pi(x)$. The conditions (6) can be also rewritten in the form

$$dx^a\nabla_a\omega^\alpha = 0, \qquad dx^a\nabla_a\pi_{\dot\alpha} = 0, \tag{7}$$

where covariant derivative $\nabla_{\alpha\dot\alpha} = \frac{\partial}{\partial x^{\alpha\dot\alpha}} + i\pi_{\dot\alpha}\frac{\partial}{\partial\omega^\alpha}$ was defined. One is covariant derivative in the *flat* twistor space. Now, parameters $\omega(x),\ \pi(x)$ can be considered as the usual twistor coordinates $Z^A = Z^A(\omega,\ \pi)$ on the twistor space T with the additional conditions

$$\nabla_{\alpha\dot\alpha}Z^A = 0. \tag{8}$$

These conditions give us an additional fibering of the coset space F, when each fiber is α – plane. The conditions (8) could be also thought as identities on any twistor space. Then every function $\Phi(Z(x))$ on a twistor space has to obey the equation $\nabla_{\alpha\dot\alpha}\Phi(Z) = 0$. So, we find the definition of global twistors and *flat* twistor spaces T.

In the same manner, the basic equations [1, 11], that define local twistor properties associated with conformal-flat space-time, arise on the coset $F' = G^{(\frac{1}{2})}/SL(2,C)$. With choice coordinates on F' via

$$F' \ni f' = \exp(\frac{i}{2}x^{\dot\alpha\alpha}P_{\dot\alpha\alpha})\exp(\frac{i}{2}\gamma^{\dot\alpha\alpha}K_{\dot\alpha\alpha})\exp(i\sigma D)\exp(i\omega^\alpha q_\alpha + i\pi^{\dot\alpha}s_{\dot\alpha}) \tag{9}$$

conditions, analogous to (6), are

$$\tilde\nabla_{\delta\dot\delta}Z = \mathcal{D}_{\delta\dot\delta}\begin{pmatrix}\omega^\alpha\\ \pi_{\dot\alpha}\end{pmatrix} + i\begin{pmatrix}0 & \delta^\alpha_\delta\delta^{\dot\beta}_{\dot\delta}\\ \mathcal{P}_{\delta\dot\delta,\beta\dot\alpha} & 0\end{pmatrix}\begin{pmatrix}\omega^\beta\\ \pi_{\dot\beta}\end{pmatrix} = 0, \tag{10}$$

where

$$\mathcal{P}_{\delta\dot\delta,\alpha\dot\alpha} = e^{2\sigma}(\partial_{\delta\dot\delta}\gamma_{\alpha\dot\alpha} + 2\gamma_{\delta\dot\alpha}\gamma_{\alpha\dot\delta}), \quad \mathcal{D}_{\alpha\dot\alpha} = e^\sigma\left(\nabla_{\alpha\dot\alpha} + \Gamma_{\alpha\dot\alpha}\right),$$

and $\Gamma_{\alpha\dot\alpha}$ are Lorentz connection in an appropriate representation, $\nabla_{\alpha\dot\alpha}$ is the same as in (7).

From Cartan's equation

$$0 \equiv d\Omega - \Omega\wedge\Omega = \left(P_a\tilde T^a{}_{bc} + K_a\tilde B^a{}_{bc} + L_{ad}\tilde R^{ad}{}_{bc}\right)dx^b\wedge dx^c, \tag{11}$$

where $\Omega = f'^{-1}df'$, particularly follows that the curvature of a conformal-flat space-time is defined by the *twistor* connection $\mathcal{P}^b_a$ as $R^{ab}_{dc} = \delta^{[a}_{[d}\mathcal{P}^{b]}_{c]} \Leftrightarrow \mathcal{P}^b_a = R^b_a - \frac{1}{6}R\delta^b_a$.

The equation, defining β – plane, can be easily found in the context of nonlinear realizations of group $G^{(1)}$, which can be gained by adding one more pair of generators $(q_{\dot\alpha},\ s_\alpha)$ with appropriate commutation relations. It is easy to check that the factor space $G^{(1)}/H$ will describe twistors $Z^A,\ \bar Z^A$ and the twistor space $T\cup\bar T$. In order to get the

standard dual relation between T and $\bar{T}$, it is obvious to use Cartan – Killing metric of $G^{(1)}$ group projected into the coset F.

Further, we will present $G^{(2)}$ extended group that related to the twistor sets $Z^A,\ W^B$ and $\bar{Z}^A,\ \bar{W}^B$. As it was demonstrated by Penrose [1] they make possible to describe massive fields and particles.

Let us introduce the following set of "twistor-like" generators

$$(q_{\alpha i},\ q_{\dot{\alpha} i}),\ (s_{\alpha i}, s_{\dot{\alpha} i}),$$

where $i = 1,\ 2$. It was an essential point that in addition to the usual conformal symmetry, from Jacoby identities we will have the "helicity charge" A and $SU(2)$ automorphism group

$$\begin{aligned} &[J_{ij}, q_l] = \tfrac{1}{2}\varepsilon_{l(i}q_{j)}, \\ &[A, q_{\alpha i}] = i q_{\alpha i}, \\ &[A, q_{\dot{\alpha}}] = -i q_{\dot{\alpha}}. \end{aligned} \tag{12}$$

Their commutation relations with the generators of the conformal group are

$$\begin{aligned} &[q_{\alpha i},\ q_{\dot{\alpha} j}] = -\varepsilon_{ij} P_{\alpha\dot{\alpha}}, \qquad [s_{\alpha i},\ s_{\dot{\alpha} j}] = \varepsilon_{ij} K_{\dot{\alpha}\alpha}, \\ &[P_{\alpha\dot{\alpha}},\ s_{\beta i}] = 2\varepsilon_{\alpha\beta} q_{\dot{\alpha} i}, \qquad [K_{\dot{\alpha}\alpha},\ q_{\beta i}] = -2\varepsilon_{\alpha\beta} s_{\dot{\alpha} i}, \\ &[P_{\alpha\dot{\alpha}},\ s_{\dot{\beta} i}] = 2\varepsilon_{\dot{\alpha}\dot{\beta}} q_{\alpha i}, \qquad [K_{\alpha\dot{\alpha}},\ q_{\dot{\beta} i}] = -2\varepsilon_{\dot{\alpha}\dot{\beta}} s_{\alpha i}, \\ &[q_{\alpha i},\ s_{\beta i}] = \varepsilon_{\alpha\beta}\varepsilon_{ij}(iD - \tfrac{3i}{2}A) + 2\varepsilon_{\alpha\beta} J_{ij} + \varepsilon_{ij} L_{\dot{\alpha}\dot{\beta}}, \\ &[q_{\dot{\alpha} i},\ s_{\dot{\beta} i}] = \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{ij}(iD + \tfrac{3i}{2}A) + 2\varepsilon_{\dot{\alpha}\dot{\beta}} J_{ij} + \varepsilon_{ij} L_{\dot{\alpha}\dot{\beta}}. \end{aligned} \tag{13}$$

The full set of commutators defining algebra of $G^{(2)}$ group consist from (2), (12) and (13).

It can be seen that subalgebra $G^{(1)} \subset G^{(2)}$ is associated with generators $(q_{\alpha 1},\ q_{\dot{\alpha} 2})$, $(s_{\alpha 2}, s_{\dot{\alpha} 1})$. Let us consider this subalgebra some more detail. We restrict our attention to some affirmations in the light of which the construction of global twistors correspondence occurs amazingly simple. Obviously, that twistors of the fundamental $SU(2,2)$ group representation having all known properties can be found in the context of nonlinear realizations of $G^{(1)}$ from trivially contracted target algebra.

We fix the coset through the following expression:

$$\exp(\frac{i}{2} P^{\dot{\alpha}\alpha} x_{\dot{\alpha}\alpha}) \exp(\frac{2i}{5} a\tilde{A}) \exp(i\omega^{\alpha} q_{\alpha 1} + i\pi^{\dot{\alpha}} s_{\dot{\alpha} 1}) \exp(i\omega^{\dot{\alpha}} q_{\dot{\alpha} 2} + i\pi^{\alpha} s_{\alpha 2}), \tag{14}$$

where $\tilde{A} = A + 2J_{12}$. On the intersection of α and β plains we have

$$\omega^{\dot{\alpha}} \pi_{\dot{\alpha}} - \omega^{\alpha} \pi_{\alpha} = 0, \tag{15}$$

which can be fulfilled then

$$\omega^{\alpha} \pi_{\beta} - \pi^{\alpha} \omega_{\beta} = \delta^{\alpha}_{\ \beta}, \quad (\alpha, \beta \to \dot{\alpha}, \dot{\beta}).$$

Notice: *It is easy to see the analogy with the harmonic space construction* [12], *where* $(\omega,\ \pi)$ *play the role of harmonical coordinates on the sphere* $S^2 = SU(2)/U(1)$. *The connection between the harmonic space construction and the algebra* $G^{(1)}$ *is available by the automorphysm* $SU(2) \subset G^{(1)}$ *subgroup.*

As this take place, real Minkowsky space coordinates can be completely "gauged away" by

$$x^{\alpha\dot{\alpha}} = -i\omega^{\alpha}\omega^{\dot{\alpha}}.$$

Conditions defining functions $\Phi(x,\omega,\bar{\omega})$ on the twistor space $T \cap \bar{T}$ are two constrains $\nabla_{(a)}\Phi = 0$, $\nabla_{\alpha\dot{\alpha}}\Phi = 0$. The solutions of the first constrain

$$\nabla_{(a)}\Phi = (\frac{\partial}{\partial a} - i\omega^{\alpha}\frac{\partial}{\partial \omega^{\alpha}} + i\omega^{\dot{\alpha}}\frac{\partial}{\partial \omega^{\dot{\alpha}}})\Phi = 0, \tag{16}$$

give us left or right helicity fields $\psi(x)$

$$\begin{aligned} n &= 0,\ 1,\ 2,\ldots \quad \rightsquigarrow \quad \Phi = e^{ina}\omega^{\alpha_1}\cdots\omega^{\alpha_n}\psi_{\alpha_1\cdots\alpha_n}(x), \\ n &= -1,\ -2,\ldots \quad \rightsquigarrow \quad \Phi = e^{ina}\omega^{\dot{\alpha}_1}\cdots\omega^{\dot{\alpha}_n}\psi_{\dot{\alpha}_1\cdots\dot{\alpha}_n}(x), \end{aligned} \tag{17}$$

where n parameterize the solutions of (16) and define homogeneous degree of the functions $\Phi(Z)$ or $\Phi(\bar{Z})$. The second constrain

$$\nabla_{\alpha\dot{\alpha}}\Phi = (\frac{\partial}{\partial x^{\alpha\dot{\alpha}}} - \pi_{\dot{\alpha}}\frac{\partial}{\partial \omega^{\alpha}} - \pi_{\alpha}\frac{\partial}{\partial \omega^{\dot{\alpha}}})\Phi = 0 \tag{18}$$

is the free equation of motion massless fields. Essentially, that on the subspace $T \cap \bar{T}$, either left or right helicity fields $\psi_{\alpha}\ldots$ can "live". It seems that the constrains (16, 18) are agreed upon conditions defined representations of the conformal group. Outlined above method can be also applied for constructing a large number of cosets $G^{(2)}/H$ by looking at the parabolic subgroups of $G^{(2)}$.

2.3 "Extended" algebra for the supertwistors

The supertwistor correspondence can be also described in the context of coset realizations of "extended" supergroups $SG^{(n|N)}$, where N denotes quantity of fermionic spinoral generators in associated superconformal group. In this section we will describe $SG^{(2|1)}$ supergroup. The algebra of the group $SG^{(2|1)}$ is obtained from relation (2, 12, 13) and $N = 1$ conformal supersymmetry

$$\begin{aligned} &\{Q_{\alpha}, Q_{\dot{\alpha}}\} = P_{\alpha\dot{\alpha}}, && \{R_{\alpha}, R_{\dot{\alpha}}\} = K_{\alpha\dot{\alpha}}, \\ &[P_{\alpha\dot{\alpha}}, R_{\beta}] = 2i\varepsilon_{\alpha\beta}Q_{\dot{\alpha}}, && [K_{\alpha\dot{\alpha}}, Q_{\beta}] = 2i\varepsilon_{\alpha\beta}R_{\dot{\beta}}, \\ &[P_{\dot{\alpha}\alpha}, R_{\dot{\beta}}] = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}Q_{\alpha}, && [K_{\alpha\dot{\alpha}}, R_{\dot{\beta}}] = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}R_{\alpha}, \end{aligned} \tag{19}$$

by adding $(Q_i,\ R_i)$ generators and charge B having odd Grassmannian parities

$$\begin{aligned} &[B,\ Q_{\alpha(\dot{\alpha})}] = \pm\tfrac{1}{2}Q_{\alpha(\dot{\alpha})}, && [B,\ Q_i] = \tfrac{1}{2}Q_i, \\ &[B,\ R_{\alpha(\dot{\alpha})}] = \mp\tfrac{1}{2}R_{\alpha(\dot{\alpha})}, && [B,\ R_i] = -\tfrac{1}{2}R_i, \\ &[A,\ Q_i] = -iQ_i, && [A,\ R_i] = iR_i. \end{aligned} \tag{20}$$

Jacoby identities fix the other relations

$$\begin{aligned}
&[q_{\alpha i},\ s_{\beta j}] = \varepsilon_{ij}\varepsilon_{\alpha\beta}(iD + B - 3i/2A) + \varepsilon_{ij}L_{\alpha\beta} + 2\varepsilon_{\alpha\beta}J_{ij},\\
&[q_{\dot\alpha i},\ s_{\dot\beta j}] = \varepsilon_{ij}\varepsilon_{\dot\alpha\dot\beta}(iD - B + 3i/2A) + \varepsilon_{ij}L_{\dot\alpha\dot\beta} + 2\varepsilon_{\dot\alpha\dot\beta}J_{ij},\\
&\{Q_i,\ R_j\} = \varepsilon_{ij}(2iB + \tfrac{1}{2}A) - iJ_{ij},\\
&\{Q_{\dot\alpha}R_{\dot\beta}\} = \varepsilon_{\dot\alpha\dot\beta}(D - 3iB + \tfrac{1}{2}A) - iL_{\dot\alpha\dot\beta},\\
&\{Q_\alpha R_\beta\} = \varepsilon_{\alpha\beta}(D + 3iB - \tfrac{1}{2}A) - iL_{\alpha\beta},\\
&\{Q_i,\ Q_{\dot\alpha}\} = q_{\dot\alpha i}, \qquad \{R_i,\ R_{\dot\alpha}\} = s_{\dot\alpha i},\\
&\{R_i,\ Q_\alpha\} = -iq_{\alpha i}, \qquad \{Q_i,\ R_\alpha\} = is_{\alpha i},\\
&[q_{\delta i},\ Q_j] = -\varepsilon_{ij}Q_\delta, \qquad [s_{\alpha i},\ R_j] = \varepsilon_{ij}R_\alpha,\\
&[q_{\alpha i},\ R_\beta] = -2\varepsilon_{\alpha\beta}R_i, \quad [s_{\alpha i},\ Q_\beta] = 2\varepsilon_{\alpha\beta}Q_i,\\
&[q_{\dot\alpha i},\ R_{\dot\beta}] = 2i\varepsilon_{\dot\alpha\dot\beta}Q_i, \quad [s_{\dot\alpha i},\ Q_{\dot\beta}] = 2i\varepsilon_{\dot\alpha\dot\beta}R_i,\\
&[q_{\dot\alpha i},\ R_j] = i\varepsilon_{ij}Q_{\dot\alpha}, \qquad [s_{\dot\alpha i},\ Q_j] = i\varepsilon_{ij}R_{\dot\alpha}.
\end{aligned} \tag{21}$$

Various versions of the equations defining left(right) and central $\alpha(\beta)$ – superplanes and the properties of both global and local supertwistors are easily determined. So, α – superplanes are defined by the following condition

$$d\omega^\alpha = dx^{\dot\alpha\alpha}\pi_{\dot\alpha} + 2d\theta^\alpha\xi, \quad d\xi = 2d\bar\theta^{\dot\alpha}\pi_{\dot\alpha}, \tag{22}$$

where parameters (ω, π, ξ) defining supertwistor components are associated with generators $(q_{2\alpha}, s_{2\dot\alpha}, R_2)$.

Because there is a formal generalization of the contour integrals method for constructing of chiral superfields, it seems that there is a local equivalence between the general conformal supermultipletes and their potentials. One may suppose, that there is some cohomological description including explicit expressions for superfields and prepotential representations.

3 Summary and further perspectives

In this paper we proposed some class of the "twistor-like" extensions $G^{(n|N)}$ for $n = 1, 2;\ N = 0, 1$ (super)conformal group and illustrated how the variants of the (super)twistor correspondence can arise on the cosets $G^{(n|N)}/H$. This approach seems us promising for more detailed analysis twistor field models. Particularly, it can be useful for the models given below.

It is well known that the obstructions of integrability of some nonlinear field models (such as self-dual gravity, YM) in the twistor approach vanish. The general solutions of these models are defined by the intrinsic properties of complex manifolds rather then by the equations of motion or by action principles. An interaction therewith is encoded in deformations of the complex structure of twistor space. The description of appropriate deformations, which are connected with whole Poincaré and (super)conformal gravity, is still an open problem.

Proposed in ref. [13] radical modification of the standard string theory to a four-dimensional field theory, where Riemann surfaces and holomorphic functions of 2D CFT are replaced by generalized twistor spaces and holomorphic "one-functions" respectively,

seems to be to have a set of remarkable properties of exactly solvable as it takes place in 2D.

A more detailed consideration of the mentioned problems will be done in subsequent publications.

References

[1] R. Penrose and W. Rindler, "Spinors and space-time", Cambridge U.P., Cambridge, (1986); R. O. Wells, *Bull. Amer. Math. Soc.* (1979), **1**, 296; R. Penrose and M. A. H. MacCallum, *Phys. Rep.* **6** (1972) 241; L. P. Hughston, "Twistors and particles", *Lecture Notes in Physics*, Berlin (1979).

[2] M. F. Atiyah, N. J. Hitchin, I. M. Singer, *Proc. R. Soc. Lond. A.*, (1978), **362**, 425; A. L. Besse, "Einstein Manifolds", Springer-Verlag, Berlin, (1987).

[3] R. S. Ward, *Phys. Lett.* **61A** (1977) 81; M. F. Atiyah, J. S. D. Jones, *Comm. Math. Phys.*, **61**, (1978), 97; Yu. I. Manin, "Gauge fields and complex geometry", Moscow, Nauka, (1984); I. V. Volovich, *Theor. Math. Phys.* **54**, (1983), 89 (in Russian).

[4] R. S. Ward, *Phil. Trans. Roy. Soc.* **A315** (1985) 451; L. Mason and G. A. J. Sparling, *Phys. Lett.* **B137** (1989) 29.

[5] E. Witten, *Nucl. Phys.* **B266** (1986) 245.

[6] A. Ferber, *Nucl. Phys.* **B132** (1978) 55.

[7] D. P. Sorokin, V. I. Tkach and D. V. Volkov, *Mod. Phys. Lett.* **A4**, (1989) 901; D. P. Sorokin, V. I. Tkach, D. V. Volkov and A. A. Zheltukhin, *Phys. Lett.* **B216** (1989) 302; D. P. Sorokin, *Fortshr. der Phys.* **38** (1990); Y. Eisenberg, *Phys. Lett.* **B233**, (1989), 349; N. Berkovits, *Nucl. Phys.* **B350** (1991), 193.

[8] W. Siegel, *Phys. Lett.* **B128** (1983), 397.

[9] P. S. Howe, hep-th 9512066.

[10] E. Sokatchev, *Phys. Lett.* **B169** (1986), 209.

[11] S. A. Merkulov, *Commun. Math. Phys.* **93**, (1984), 325.

[12] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, *Ann. Phys.* (N.Y.) **185**, (1988), 1; M. Evans, F. Gursey and V. Ogievetsky, *Phys. Rev.* **D47**, (1993), 3496; C. Devchand, V. Ogievetsky, *Phys. Lett.* **B297**, (1992), 93.

[13] A. P. Hodges, R. Penrose and M. A. Singer, *Phys. Lett.* **B216**, (1989), 48.